

# ON THE GREEN-NAGHDI THEORY OF MIXTURES

R. I. Nigmatulin

We examine some aspects of [1] and show the nonapplicability of some of these to the study of heterogeneous mixtures, which does not exclude the possibility of their application to the description of homogeneous mixtures (mixtures of gases, solutions, alloys, and so on).

In [1] there are written the energy equations and the inequalities for the entropy of each of  $n$  mixture components which are in mutual relative motion. On the basis of the requirement that these relations be invariant with the imposition of rigid motions on the mixture as a whole, limitations are obtained on the expressions characterizing the interaction of these  $n$  components. This part of the paper relates to any mixtures (including polyphase) which can be described by the motion of  $n$  continua. Then the temperature and the other thermodynamic functions for the mixture are introduced and on the basis, it would appear, of general thermodynamic assumptions (in the case of two components) relative to the internal energies  $U_1$ ,  $U_2$  and the pressures  $p_1$ ,  $p_2$  of the components

$$\begin{aligned} U_1 &= U_1(\rho_1, \rho_2, T_1, T_2), & U_2 &= U_2(\rho_1, \rho_2, T_1, T_2), \\ p_1 &= p_1(\rho_1, \rho_2, T_1, T_2), & p_2 &= p_2(\rho_1, \rho_2, T_1, T_2) \end{aligned} \quad (1)$$

where  $\rho_i$ ,  $T_i$  are the average density (mass of the  $i$ -th component per unit mixture volume) and temperature of the  $i$ -th component ( $i = 1, 2$ ), the energy equation and the inequality for the entropy of the mixture as a whole are obtained. Here the relations between the thermodynamic functions of the mixture and the components in the general case depend on the temperature history and the kinematic variables. In the particular case in which we assume additivity of the internal energy and entropy of the mixture over the component masses

$$\rho U = \rho_1 U_1 + \rho_2 U_2, \quad \rho S = \rho_1 S_1 + \rho_2 S_2 \quad (2)$$

the condition  $T_1 = T_2 = T$  must be satisfied.

But the assumptions (1) are valid only for homogeneous mixtures. For heterogeneous mixtures, in which there are surfaces where some of the parameters undergo discontinuities (mixture of gas or liquid with particles, mixtures of a liquid with bubbles, and so on) the parameters  $\rho_1$ ,  $\rho_2$ ,  $T_1$ ,  $T_2$  are not sufficient to characterize the state of each phase. Therefore, in the theory of interpenetrating motion of polyphase media there are for each phase the quantities  $\alpha_i$  (volumetric content of the  $i$ -th phase), which show the fraction of the mixture volume occupied by the  $i$ -th phase. For example, in a gas-particle mixture in which the particle dimensions are quite large, in order to be able to neglect surface effects both (2) and the following expression (see [2]) must hold

$$p_1 = p_2 = p(\rho_1^\circ, T_1) \quad (\rho_1^\circ = \rho_1 / \alpha_1) \quad (3)$$

where  $p(\rho_1^\circ, T_1)$  is the equation of state for the pure gas. Thus, in the equations of state of the phases there appear the true phase densities  $\rho_1^\circ = \rho_1 / \alpha_1$ , and the assumption on additivity of the internal energy and the entropy of the mixture with respect to the phase masses (2) involves the possibility of neglecting the influence of the surface layer, which can be done when the inclusion dimensions exceed by many times the intermolecular distances. Moreover, the quantities  $\alpha_i$  appear in the equations of motion of the phases. The corresponding thermodynamic analysis, in which the concept of mixture temperature does not appear, was made for several cases in [2, 3].

---

Moscow. Translated from *Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki*, Vol. 11, No. 3, p. 168, May-June, 1970. Original article submitted February 4, 1970.

© 1973 Consultants Bureau, a division of Plenum Publishing Corporation, 227 West 17th Street, New York, N. Y. 10011. All rights reserved. This article cannot be reproduced for any purpose whatsoever without permission of the publisher. A copy of this article is available from the publisher for \$15.00.

#### LITERATURE CITED

1. A. E. Green and P. M. Naghdi, "A theory of mixtures," *Archive for Rational Mechanics and Analysis*, 24, No. 4 (1967), pp. 243-263.
2. R. I. Nigmatulin, "Some relations of nonequilibrium thermodynamics for a two-temperature and two-velocity gas with phase changes," *Izv. AN SSSR, MZhG*, 3, No. 5 (1968).
3. P. P. Zolotarev and V. N. Nikolaevskii, "Thermodynamic analysis of unsteady processes in deformable porous media saturated with liquid and gas," *Collection: Theory and Practice of Oil Extraction [in Russian]*, Nedra, Moscow (1966).